Liquidity Commonality Across the Bond and CDS Markets

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The standard asset pricing models are usually built on the assumptions of no arbitrage and the frictionless market. However, illiquidity exists in various asset markets. Studies, such as Acharhya and Pedersen [2005] and Korajczyk and Sadka [2007], show that liquidity plays an important role in asset pricing. Most previous liquidity studies are restricted to the stock market. Some articles, such as Amihud [2002], examine how to measure the liquidity, while others, such as Hasbrouck and Seppi [2001] and Huberman and Halka [2001], explore the commonality in various equity liquidity measures. There is also a growing literature of corporate bond liquidity (see Chen, Lesmond, and Wei [2007], Downing, Underwood, and Xing [forthcoming], and Bessembinder, Maxwell, and Venkataraman [2006]) that shows that liquidity could explain a significant part of bond returns.

However, the study of commonality in the illiquidity across different markets is rather limited. Chordia, Sarkar, and Subrahmanyam [2005] study the liquidity commonality between the stock and Treasury bond markets and find that shocks to stock and Treasury bond market liquidity and volatility are significantly correlated, which suggests that a common factor drives liquidity and volatility in these markets. However, there is no study that combines the information from various liquidity measures both in the credit derivatives and corporate bond markets. This article tries to fill this gap by exploring the properties of the liquidity measures across the corporate bond and CDS markets.

In addition, it is meaningful to examine whether a common or systematic liquidity factor affects the credit spreads, which may shed some lights on the pricing of risky bonds. Empirical tests of structural models, such as Collin-Dufresne, Martin, and Goldstein [2001] and Huang and Huang [2003], show that the default risk factors only explain a small fraction of observed credit spreads in the market. Several studies show that the illiquidity in the corporate bond market may explain the basis between the CDS and bond spreads. Longstaff, Mithal, and Neis [2005] show that the difference between the CDS and bond spreads, proxied as the non-default component, is strongly related to measures of bond-specific illiquidity. Han and Zhou [2006] construct different bond market liquidity measures to show the significant impacts on the non-default component in the bond yield spreads over the Treasury risk-free yields. The findings in this article show the existence of a strong common liquidity factor across the bond and CDS markets, which plays an important role in pricing risky debt.

The article constructs two credit market liquidity measures from the credit default swap data and seven bond market liquidity measures of trading frequency, trading costs, and trading prices from the bond transaction data.
We combine the liquidity information from various measures to form a common facet of asset liquidity in the fixed-income market. The credit market liquidity measures are proportion of zero daily spread changes (Szero) and market depth (Depth), respectively. For the bond liquidity measures, the article employs three category measures. The first category is related to trading frequency, which includes number of total trades in a month (Ntrad) and number of days with at least one transaction in a month (Nday). The second category is related to trading costs and includes the effective bid-ask spread estimated from Roll's [1984] model (Bidask) and the inter-quartile price changes (IQR). The third category is related to trading frequency and includes the Amihud [2002] measure, the square root of the Amihud measure (S.Amihud), and the range (Range). We examine the liquidity characteristics in both the investment-grade and high-yield firms. High-yield bond investors tend to trade more frequently, which is consistent with the high trading frequency measures (Ntrad and Nday). However, the investment-grade sample shows higher levels of liquidity than the high-yield sample in all the other categories except those related to trading frequency.

The study uses the asymptotic principal component (APC) method of Connor and Korajczyk [1986] to extract the common factors from each individual liquidity measure and across different measures. The method is also employed in Korajczyk and Sadka [2007], which explores the commonality and pricing information in a comprehensive set of equity liquidity measures across assets. The results of factor decompositions show a strong liquidity commonality in the fixed-income markets. Overall, the cross-sectional average R-square from the time-series regressions of each liquidity measure on its first three common factors is in the range of 20% to 87%. Moreover, the factors extracted across all the measures also show strong commonality. The cross-sectional average R-square from the time series regressions of each liquidity measure on the first three common factors extracted from all the liquidity measures is higher than those from the time-series regressions of common factors extracted within individual liquidity measures. In addition, a pair-wise correlation analysis between the liquidity measures suggests that the various liquidity measures are strongly correlated. The results suggest that there is a systematic liquidity factor in the corporate bond market.

To explore the pricing information provided by the common liquidity factors identified in various liquidity measures, this study tries to examine whether the unexplained portion in the credit spreads, which could not be attributed to the default risk factors from the traditional structural models, would be affected by the liquidity risk. The article estimates the unexplained portion of the credit spreads by both linear and nonlinear regressions. The linear regression uses the default-related variables implied by the Merton [1974] model. The non-linear regression method is similar to that of Acharya and Johnson [2007], who studied of insider information in credit derivative market. The residuals from the regressions are viewed as the component in the credit spreads that is not explained by factors suggested by traditional structural models.

The article shows that the common liquidity factor, extracted from various individual measures, has a strong impact on the unexplained portion of credit spreads by default factors. Since we have seen a strong commonality in the liquidity measures, the article runs the monthly panel regressions of the unexplained portion in credit spreads on the first, the second, and the third common factors extracted from the liquidity measures. If the systematic liquidity factor were priced into the credit spreads, these factors should be significant. The results are consistent with the argument, and the common factors, which are extracted across all the various liquidity measures, are highly significant for the unexplained portion in credit spreads, both in the investment-grade sample and in the high-yield sample. The significance in the investment-grade firms is higher than that in the high-yield firms. The finding is consistent with previous studies, such as Jones, Mason, and Rosenfeld [1984] and Huang and Huang [2003], which show that the standard structural models of credit risk predict lower credit spreads than observed market spreads for corporate risky debt, especially for investment-grade bonds. In addition, it should be noticed that the significance level does not decrease from the first to the third common factors, which suggests that the liquidity risk factor is quite persistent.

**LITERATURE REVIEW AND MOTIVATION**

Market liquidity is the ease of trading a security, which is usually ignored in the traditional asset pricing models. Illiquidity could come from high transaction costs (brokerage fees, commissions, etc.), demand pressure and inventory risk in the market, private information, search friction, or short-sale constraints. A perfectly liquid market does not exist in reality, which suggests that liquidity risk
is priced in the asset prices to a certain degree. Some articles provide theoretical framework to show how liquidity impacts financial market prices, such as Amihud and Mendelson [1986]. Some studies find that liquidity could help predict expected returns in the time series (e.g., Amihud [2002]) and expected equity returns are cross-sectionally related to liquidity risk (e.g., Pastor and Stambaugh [2003]). Acharya and Pedersen [2005] show that a security’s return depends on its expected liquidity as well as on the covariance of its own return and liquidity in the liquidity-adjusted capital pricing model.

Most previous liquidity studies focus on the equity market and the cross-sectional determinants of liquidity, such as Benston and Hagerman [1974] and Stoll [1978]. Recently, some work has examined the time-series properties of the liquidity either in the equity market or in the U.S. Treasury bond market. A few studies show there is commonality in trading activity and equity liquidity, such as Hasbrouck and Seppi [2001] and Chordia, Roll, and Subrahmanyan [2000, 2001]. Korajczyk and Sadka [2007] show that there is strong commonality among different equity liquidity measures and the common factors have pricing information. For the Treasury bond market, some studies, such as Fleming [2003], Balduzzi, Elton, and Green [2001] and Brandt and Kawajecz [2002], examine the time series of liquidity measures and study the relation between liquidity, order flow, the yield curve, returns, and other characteristics. However, there are limited studies that examine the liquidity across different security markets. Chordia, Sarkar, and Subrahmanyan [2005] examine the liquidity dynamics across the stock market and the U.S. Treasury bond market. They find that shocks to stock and Treasury bond market liquidity and volatility are significantly correlated.

This article studies the liquidity commonality in the credit derivatives and corporate bond markets, which has not been examined in the literature. Due to the fast development of credit derivatives market, in which credit default swaps are the most heavily traded credit derivative instruments, it is necessary to explore the liquidity in the CDS market and the associated corporate bond market. Although some articles examine the spillover effects of the liquidity from other markets to the CDS spreads, such as Tang and Yan [2006], there is no detailed examination of the liquidity commonality in the credit derivative and corporate bond markets.

Using the Trade Reporting and Compliance Engine (TRACE) introduced by the FINRA (Financial Industry Regulatory Authority), formerly the National Association of Securities Dealers (NASD), this article constructs bond liquidity measures directly from the intraday transaction data, which keeps the same spirit as those equity liquidity measures from intraday data in the microstructure literature. Then the article does not need to rely on indirect measures such as those used in previous studies, such as coupon rate (e.g., Gehr and Martell [1992], Longstaff, Mithal, and Neis [2005]), total amount of a bond issue (e.g., Alexander, Edwards, and Ferri [2000] and Hong and Warga [2000]), bond age (e.g., Alexander, Edwards, and Ferri [2000], Hong and Warga [2000], and Elton, Gruber, Agrawal, and Mann [2001]). This study extracts the common components from various liquidity measures across the credit and corporate bond markets and examines the extent of commonality across the measures.

Structural models of default risk are developed from the framework of Black and Scholes [1973] and Merton [1974], in which the corporate debt is viewed as a portfolio composed of risk-free debt and a short position in a put option on the underlying firm value. According to the structural models, the credit spreads are determined by default risk variables, such as asset volatility, firm leverage, and the term structure of interest rates. Along the line, numerous studies have extended the traditional Merton model by incorporating other economic considerations. Longstaff and Schwartz [1995] propose a framework with a stochastic interest rate process. Some studies, such as Anderson and Sundaresan [1996], incorporate strategic debt service into the premium on risky corporate debt. Other studies relax the assumption of exogenously determined default boundaries in the Merton model (Leland and Toft [1996]) or make all the firms have flexible capital structures corresponding to firm value changes (Collin-Dufresne and Goldstein [2001]). However, the empirical studies do not find supportive results for these models. Jones, Mason, and Rosenfeld [1984] find that the corporate credit spreads, generated by the Merton [1974] model, are significantly below the credit spreads in the market. Collin-Dufresne, Goldstein, and Martin [2001] find that variables that should theoretically determine credit spread changes actually have only about 25% explanatory power. Huang and Huang [2003] argue that default risk only partially accounts for the observed credit spreads, leaving the rest again unexplained by the variables implied from the structural models of credit risk.1

Recently, several studies suggest that liquidity premium helps explain part of the component in credit
spreads that could not be attributed to default risk factors. Longstaff, Mithal, and Neis [2005] use CDS data to estimate direct measures of default and nondefault components in corporate bond yields. Their results show that the nondefault component in the credit spreads is time varying and strongly linked to measures of bond-specific liquidity. Chen, Lesmond, and Wei [2007] find that liquidity plays an important role in corporate bond valuation after controlling for bond-specific, firm-specific, and macroeconomic variables. However, these studies have not shown whether there is a common or systematic liquidity factor in the fixed-income market and the relation between the liquidity common factor and the nondefault component in the credit spreads. Motivated by the previous literature, this article studies whether there exists a strong liquidity common factor and whether the liquidity risk factor could explain the component in the credit spreads that could not be attributed to the default risk factors in the traditional structural models.

THE DATA AND LIQUIDITY MEASURES

The sample has 192 U.S. non-financial firms from the sample period from July 2002 to December 2005. The credit default swap data are from Markit, which has a strict procedure to collect daily quotes of CDS spreads from major dealers in the credit derivatives market. The bond price data are from TRACE (Trade Reporting and Compliance Engine). There are 93 investment-grade firms with ratings above and equal to BBB and 99 non-investment-grade firms (BB, B, CCC). The leverages are computed as the ratio of total debt to the sum of total debt and market capitalization. The debt is computed as the sum of long-term debt (data51, Compustat Quarterly) and the current liabilities in debt (data45, Compustat Quarterly). The mean leverage across the sample is 0.40, and the highest leverage occurs in the high-yield sample.

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**EXHIBIT 1**

Descriptive Statistics of the Sample, July 1, 2002 to December 31, 2005

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-Year CDS Spreads (bps)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>213.50</td>
<td>136.84</td>
<td>17.77</td>
<td>1913.98</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>77.85</td>
<td>62.39</td>
<td>19.21</td>
<td>377.57</td>
</tr>
<tr>
<td>High Yield</td>
<td>340.93</td>
<td>276.36</td>
<td>43.31</td>
<td>1913.98</td>
</tr>
<tr>
<td>Size (000,000,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>12.84</td>
<td>6.65</td>
<td>0.22</td>
<td>221.64</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>21.63</td>
<td>12.61</td>
<td>1.20</td>
<td>221.64</td>
</tr>
<tr>
<td>High Yield</td>
<td>4.59</td>
<td>2.87</td>
<td>0.22</td>
<td>21.27</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.40</td>
<td>0.38</td>
<td>0.04</td>
<td>0.92</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>0.29</td>
<td>0.26</td>
<td>0.04</td>
<td>0.79</td>
</tr>
<tr>
<td>High Yield</td>
<td>0.50</td>
<td>0.49</td>
<td>0.10</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Notes: Of 192 non-financial firms, there are 93 investment-grade firms (AAA, AA, A, and BBB) and 99 non-investment-grade firms (BB, B, CCC). Market capitalization (size) is the product of stock prices and outstanding number of shares, which is measured in billions of dollars. For each obligor, the mean of its five-year CDS spreads and size is computed, and then the descriptive statistics are calculated across the sample.
bonds that each firm has in the sample is 8. The reporting
scheme of TRACE facilitates the mandatory reporting and
provides increased price transparency on an immediate
basis to market investors and regulators in corporate bonds.

In addition, the Treasury yields are used in the fol-
lowing empirical tests, which are obtained from the Fed-
eral Reserve Bank.

**CDS Liquidity Measures**

The study constructs the monthly time series of two
credit market liquidity measures for each firm. One is the
proportion of zero daily spread changes (Szero), which is
computed as the ratio of zero daily spread changes among
all the non-missing observations in a month. The larger pro-
portion of zero daily spread changes implies low liquidity
of the CDS obligator. The proportion of zero spread changes
is similar to the proportion of zero returns, which is an equity
liquidity measure used in Lesmond, Ogeden, and Trzcinka
[1999]. Markit also provides the number of contributors for
the daily five-year CDS composite quotes, which are usu-
ally large investment banks or dealers in the credit deriva-
tives market. Thus, the other CDS market liquidity measure
is market depth (Depth), which is calculated as the average
number of contributors who provide daily quotes of five-
year CDS spreads to Markit in a month. A larger number
of contributors mean higher liquidity for the obligor.

Exhibit 2 describes the statistics of the CDS market
liquidity measures. The study computes the average of
the measures for each firm from the monthly time series.

Then the descriptive statistics are computed across the
sample. The mean of Szero for the investment-grade
sample is lower than that for the high-yield sample. The
mean of Depth for the investment-grade sample is higher
than that for the high-yield sample. As shown in Exhibit 2,
the investment-grade firms have higher levels of liquidity
than the high-yield firms.

**Bond Liquidity Measures**

The study constructs monthly time series of seven
bond market liquidity measures, which are related to
trading frequency, trading costs, and trading prices. Some
of the measures are also commonly used in the stock liq-
uidity literature, such as Hasbrouck [2006]. The monthly
measures for each bond are computed in the sample. Then
the measures for a firm are computed as the average of the
monthly measures across all the bonds of that firm, which
is the issuer of its bonds.

Two measures are related to trading frequency.
Number of trades (Ntrad) is measured as the number of
total trades occurred in a month. As shown in Exhibit 3,
the mean of Ntrad is 54 for the whole sample. The average
Ntrad of the investment-grade sample (45) is lower than
that of the high-yield sample (62). In addition, the stan-
dard deviation of Ntrad for the high-yield firms is over
1.5 times larger than that for the investment-grade firms.

Number of days (Nday) is measured as the number of
days with at least one trade in a month. As shown in
Exhibit 3, the mean of Nday for the investment-grade

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**Exhibit 2**  
Descriptive Statistics of the CDS Market Liquidity Measures

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>Std.Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Proportion of Zero Daily Spread Changes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.15</td>
<td>0.13</td>
<td>0.00</td>
<td>0.59</td>
<td>0.11</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>0.10</td>
<td>0.10</td>
<td>0.00</td>
<td>0.47</td>
<td>0.08</td>
</tr>
<tr>
<td>High Yield</td>
<td>0.19</td>
<td>0.16</td>
<td>0.01</td>
<td>0.59</td>
<td>0.12</td>
</tr>
<tr>
<td><strong>CDS Market Depth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>10</td>
<td>10</td>
<td>3</td>
<td>19</td>
<td>4</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>12</td>
<td>12</td>
<td>4</td>
<td>18</td>
<td>2</td>
</tr>
<tr>
<td>High Yield</td>
<td>8</td>
<td>7</td>
<td>3</td>
<td>19</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes: Proportion of zero spread changes (Szero) is the ratio of zero daily spread changes to the total number of non-missing observations in a month. Market
depth (Depth) is measured as the average number of contributors for five-year CDS quotes in a month. For each firm, the average of the measures is calculated,
then the descriptive statistics are computed across the sample.
Notes: The monthly measures for each bond in the sample are computed. Then the measure for the firm is computed as the mean of the monthly measures across all the bonds of a firm. The following statistics are computed from the monthly measures for all the firms in the sample (July 2002 to December 2005). There are two measures related to trading frequency. Ntrad is defined as the number of total trades in a month. Nday is defined as the number of days with at least one trade in a month. There are two measures related to trading costs. Bidask is estimated from the Roll [1984] model, which is computed as:

\[ \text{Bidask} = 2 \sqrt{-\text{Cov}(\hat{p}_{it}, \hat{p}_{it})} \]

and \( \hat{p} = \log(p) \). IQR is computed as:

\[ \text{IQR} = \frac{Q_{25} - Q_{75}}{100} \]

There are three measures related to trading prices. Amihud is measured as:

\[ \text{Amihud} = \frac{1}{Q_{it}} \left( \frac{1}{Q_{it}} \right) \]

where \( Q_{it} \) is the dollar volume of a trading size (in million dollars) computed as the product of the trading price and volume. S.Amihud is the square root of the monthly Amihud measure. Range is computed as:

\[ \text{Range} = \frac{\max(Q_{it}) - \min(Q_{it})}{Q_{it}} \]

The Appendix contains the definitions of the other measures.
sample is similar to that for the high-yield sample. The statistics of trading frequency measures shows that the high-yield bonds are more traded than investment-grade bonds.

Two measures are related to trading costs. Effective bid–ask spreads (Bidask) is estimated by Roll's [1984] model as

$$\text{Bidask} = 2\sqrt{\text{Cov}(\hat{p}_{i,t}, \hat{p}_{i,t} - \hat{p}_{i,t-1})}$$

where $\hat{p}$ is the logarithm of bond prices. Consistently large effective bid–ask spreads imply the bond trading is less liquid. The study uses all the trades each day to estimate the effective bid–ask spreads and compute the average value in a month as the effective bid–ask spreads. If a bond has less than six observations on a trading day, then the bid–ask measure is missing for that day. As shown in Exhibit 3, the mean of Bidask is $0.82$ per par bond for the whole sample. The investment-grade sample has a smaller Bidask than the high-yield sample. The inter-quartile range of traded prices (IQR) is measured as the price differences between the 75th percentile and 25th percentile divided by the average price in a day. The monthly measure is computed as the average of the daily values denoted in percentage.

$$\text{IQR} = \frac{p_{i,75th} - p_{i,25th}}{p_i} \times 100$$

Exhibit 3 shows that the mean of the investment-grade sample is lower than that of the high-yield sample. The statistics of trading cost measures suggests that the investment-grade firms have a higher liquidity level associated with trading costs.

Three measures are related to trading prices. The Amihud illiquidity (Amihud) measure is frequently used in the stock market liquidity literature, such as Amihud [2002] and Hasbrouck [2006]. The Amihud measure is computed as the ratio of absolute price change percentage to the dollar volume. Then the monthly measure is the mean of all daily values in a month.

$$\text{Amihud} = \left| \frac{p_{i,t} - p_{i,t-1}}{p_{i,t-1}} \right| \times \frac{Q_{i,t}}{Q_{i,t-1}} \times 100$$

where $Q_i$ is the dollar volume of a trading size (in million dollars) computed as the product of the trading price and volume. A large Amihud value suggests low liquidity.

The monthly square root of the Amihud measure (S.Amihud) is the square root of the Amihud measure. As shown in Exhibit 3, the mean of the Amihud for the investment-grade firms is much lower than that of the high-yield firms. For the square root of Amihud measures, the mean values of the investment-grade and high-yield samples are in the similar magnitude. The maximum value of square root of Amihud measures occurs in the high-yield sample and the minimum value occurs in the investment-grade sample. The measure, Range, is defined as the ratio of daily price range standardized by daily mean price to the dollar volume, which is also used in Downing, Underwood, and Xing [forthcoming].

$$\text{Range} = \frac{\max(p_{i,t}) - \min(p_{i,t})}{\bar{p}_i} \times 100/Q_i$$

where $Q_i = \Sigma Q_{i,t}$ is sum of the dollar volume in a day. As shown in Exhibit 3, the mean of the investment-grade firms is much lower than that of the high-yield firms. The liquidity measures related to trading prices show that the investment-grade firms have a higher level of liquidity than the high-yield firms.

After constructing the credit and bond liquidity measures, the study has an unbalanced panel of nine CDS and bond liquidity measures for 192 firms. There are 42 monthly time series (July 2002–December 2005) if there is no missing value for a firm. In addition, the study examines the correlations among the individual liquidity measures in Exhibit 4. The Pearson correlation is computed from the time series of cross-sectional mean of individual liquidity measures. As shown in Exhibit 4, most of the measures are highly correlated, although the price-related bond liquidity measures are the least related with other measures.

**FACTOR DECOMPOSITION OF LIQUIDITY MEASURES**

To examine whether there is commonality across firms in the fixed-income market for each liquidity measure, the study does the factor decomposition for each individual measure and computes how much variation (R-square) in each liquidity measure could be explained by the first three extracted common factors. The method follows that of Korajczyk and Sadka [2007], which analyzes the commonality across alternative stock market liquidity measures.
**Exhibit 4**

Pearson Correlations between the Individual Liquidity Measures

<table>
<thead>
<tr>
<th></th>
<th>Szero</th>
<th>Depth</th>
<th>Ntrad</th>
<th>Ndays</th>
<th>Bidask</th>
<th>Iqr</th>
<th>Amihud</th>
<th>S.Amihud</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>-0.85</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ntrad</td>
<td>0.91</td>
<td>-0.81</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ndays</td>
<td>0.85</td>
<td>-0.68</td>
<td>0.91</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bidask</td>
<td>0.89</td>
<td>-0.69</td>
<td>0.81</td>
<td>0.76</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iqr</td>
<td>0.80</td>
<td>-0.54</td>
<td>0.76</td>
<td>0.76</td>
<td>0.87</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amihud</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.10</td>
<td>-0.22</td>
<td>-0.01</td>
<td>-0.07</td>
<td>1.00</td>
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<tr>
<td>S.Amihud</td>
<td>0.55</td>
<td>-0.29</td>
<td>0.45</td>
<td>0.44</td>
<td>0.67</td>
<td>0.66</td>
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<tr>
<td>Range</td>
<td>-0.14</td>
<td>0.06</td>
<td>-0.26</td>
<td>-0.39</td>
<td>-0.17</td>
<td>-0.23</td>
<td>0.95</td>
<td>0.37</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: The liquidity measures analyzed are proportion of daily spread changes (Szero), depth, number of trades in a month (Ntrad), number of days with at least one trade in a month (Nday), effective bid–ask spread (Bidask) estimated from Roll’s [1984] model, the inter-quartile range (IQR), defined as the ratio of the difference between 75th percentile and 25th percentile in a day to the average price of that day, the Amihud [2002] measure, the square root of the Amihud measure (S.Amihud), and range, defined as the ratio of daily price range normalized by average price in a day to the sum of dollar volume in that day.

Because there are nine measures with different units, the study first standardizes each measure by the sample mean and standard deviation of the cross-sectional average liquidity measures, which are computed by all the non-missing data prior to month t. Let $\bar{L}_{it}$ be the $n \times T$ matrix for one liquidity measure $i$, where $n$ is the number of firms and $T$ is the number of time series (42 months). At each month $t$, the study computes the mean ($\mu_{i,t}$) and standard deviation ($\sigma_{i,t}^2$) of all the data across the firms up to month $t - 1$ for the measure. Then the measure is standardized as

$$L'_{jt} = (\bar{L}_{jt} - \mu_{i,t})/\sigma_{i,t}$$

(5)

At least three months of data are required to ensure the variety in the measures, so the time series length of each measure for each firm after the standardization becomes 39 months. To keep consistent among liquidity measures with different units, the factor decomposition is performed based on these standardized measures.

Following Connor and Korajczyk [1986], the study employs the asymptotic principal components (APC) method to do the factor decomposition. The method first constructs $\Omega$ ($T \times T$ matrix) as $\Omega = \frac{LL'}{n}$ if there are no missing observations in the panel, then $n$-consistent estimates of the latent factors can be computed by calculating the eigenvectors corresponding to the $k$ largest eigenvalues of $\Omega$. Since the liquidity panel data in the study is unbalanced with missing observations, the study calculates each element of $\Omega$ by averaging over the non-missing data, following Korajczyk and Sadka [2007]. For one liquidity measure $L$ the missing observation is replaced by zero. Then the matrix $N^{*}(n \times T)$ is defined as the following: $N^{*}_{jt} = 1$ if $L_{jt}$ is available and $N^{*}_{jt} = 0$ if $L_{jt}$ is missing. Then we compute

$$\Omega^{*}_{it} = \frac{(L' \Omega)_{it}}{(N'N^{*})_{it}}$$

(6)

where $\Omega^{*}$ is corresponding to the original $\Omega$ with no missing elements and the latent factors are calculated from matrix $\Omega^{*}$. In the dataset, the constructed $\Omega^{*}$ is positive definite for each measure after the statistic check. Then the latent factors are estimated by computing the eigenvectors for the $k$ largest eigenvalues of $\Omega^{*}$, where $k = 1, 2, 3$ corresponds to the first three common principal components. We also choose the sign of the statistically estimated factors to represent liquidity, which avoids the sign indeterminacy in the factor decomposition.

For each of the nine liquidity measures, the first three principal components are extracted by using the method as previously described. If there is commonality across the sample for the liquidity measure, then the principal components could explain a certain degree of variation in the liquidity measure across different firms. To test the hypothesis, the study runs the time-series regressions of each liquidity measure for each firm on the first, the first two, and the first three principal components.
Then the cross-sectional average $R^2$ is examined. The regressions take the form

$$L^i_{jt} = \beta^i \hat{F}^i_t + \epsilon^i_{jt} \quad (7)$$

where $\hat{F}^i_t$ is the $k \times 1$ vector of the extracted common factors for month $t$. Exhibit 5 reports the cross-sectional average $R^2$ for the time-series regressions of each firm's individual liquidity measure on the first, the first two, and the first three principal components. If there are less than 15 observations of one liquidity measure for a firm, then the regression is not performed. The average number of cross-sectional firms is 169 among all the nine measures. The CDS measures have the least missing data, so their numbers of cross-sectional firms are 191. The measures related to trading frequency and trading prices have an average of 180 firms in the cross-sections. The measures relating to trading costs have the least numbers of cross-sectional firms. The results indicate that there is commonality across the sample for these liquidity measures. The strongest commonality is observed for Depth, which has an average $R^2$ of 87% for regressions on the first three principal components. The second-strongest commonality occurs for the proportion of zero daily spread changes (Szero), number of days with at least one trade in a month (Nday), and number of trades in a month (Ntrad). The average $R^2$ of the first three factors regressions for these measures is over 40%. For other measures, the three-factor model $R^2$s are all over 20%. The results show that the CDS market liquidity measures have the highest level of commonality. The bond market liquidity measures have lower level of commonality compared with the credit liquidity measures although there does exist strong commonality. Among them, the measures related to trading frequency have the highest level of commonality, while the measures related to trading prices have the lowest level of commonality.

The findings are consistent with the results of Korajczyk and Sadka [2007], Chordia, Roll, and Subrahmanyan [2000], and Hasbrouck and Seppi [2001], who find commonality among different stock market liquidity measures. In addition, Korajczyk and Sadka [2007] also find that the measures of price impact present the lowest level of commonality among the comprehensive set of stock market liquidity measures.

**Factors Across Measures**

In addition to the factor decomposition for each individual liquidity measure, the study also does principal component analysis across the two CDS liquidity measures, the seven bond liquidity measures, and all the nine liquidity measures in the fixed-income markets. The method is the asymptotic principal component (APC) with the same way to deal with missing data. The across-measure matrix $M$ is stacked by different measures as $M = [L^1, ..., L^i]$, where $i = 2$ for the CDS across measure matrix, $i = 7$ for the bond across measure matrix, and $i = 9$ for the matrix including all the liquidity measures. For the credit market liquidity, the first three principal components are extracted across the two measures. For the bond market liquidity, the first three principal components are extracted across the seven measures. Then across all nine measures, the first three principal components are also extracted.

Similar to the previous analysis, the study runs the time-series regressions of each liquidity measure for each firm on the first, the first two, and the the first three principal components extracted across all nine measures. Then the cross-sectional average $R^2$s are examined. The regressions take the form

$$L^i_{jt} = \beta^i \hat{F}^i_t + \epsilon^i_{jt} \quad (8)$$

where $\hat{F}^i_t$ is the $k \times 1$ vector of the extracted common factors extracted across all the measures for month $t$. Exhibit 6 reports the cross-sectional average $R^2$ for the time-series regressions of each firm's individual liquidity measure on the
first, the first two, and the first three principal components extracted across all the liquidity measures. The average $R^2$s are higher than those obtained from the regressions on the individual liquidity principal components, especially for the liquidity measures related to trading price (Amihud, square root of Amihud, and range). The results corroborate the existence of a strong liquidity common factor across the corporate bond and CDS markets.

Persistence of Liquidity

Besides the strong commonality in the liquidity measures, it is necessary to investigate whether the changes in liquidity are long-lasting. We calculate the autocorrelation structure of the principal common factors extracted within individual liquidity measures and across various measures. Exhibit 7 presents the autocorrelation of the first principal component extracted from each individual liquidity measure, from two credit market liquidity measures, from the bond market liquidity measures, and from all nine liquidity measures. Most liquidity factors exhibit significant autocorrelations, especially the liquidity factors extracted across measures.

However, the bond liquidity measures relating to trading prices do not exhibit much autocorrelation, particularly the Amihud measure. This is consistent with the results in Exhibit 5, where these measures present lower commonality across the sample compared with other individual measures. In addition, these measures have lower correlations with other measures, as shown in Exhibit 4. Although these individual measures present weak autocorrelations, the autocorrelation of the first principal component across all the measures is very strong. The persistence of these trading price related measures may have been incorporated in the across-all-measure liquidity common factors.

Liquidity Impact on the Unexplained Portion of Credit Spreads

In this section, the article examines whether the liquidity common factor could help explain the component of the credit spreads that cannot be attributed to default-risk factors implied from structural models, which is documented by a number of empirical studies, such as Collin-Dufresne, Goldstein, and Martin [2001] and Huang and Huang [2003].

The study uses the monthly data to run the linear regression of the CDS spread changes on the variables implied from the structural models, which include equity returns, changes in leverage, changes in equity volatility, changes in 5-year Treasury yields, changes in the slope of the yield curve (slope is the difference between 10-year and 2-year Treasury yields), market excess return, small minus big factor, high minus low factor, and changes in VIX. The regression takes the form

$$
\Delta \text{CDS} = \alpha + \beta_1 \text{ret} + \beta_{21} \Delta \text{lev} + \beta_{31} \Delta \text{vol} + \beta_{41} \Delta \text{yield} + \beta_{51} \Delta \text{slope} + \beta_{61} \Delta \text{mkt} + \beta_{71} \Delta \text{smb} + \beta_{81} \Delta \text{hml} + \beta_{91} \Delta \text{vix} + \epsilon
$$

where the variables are often used in the empirical tests of structural models. The variables ret, $\Delta$lev, $\Delta$vol, $\Delta$yield,
∆slope, and ∆vix were used by Collin-Dufresne, Goldstein, and Martin [2001]. The Fama–French factors are found to be able to explain part of the credit spread that is not accounted for by expected default and taxes in Elton, Gruber, Agrawal, and Mann [2001]. From previous analysis, we know there is a strong commonality across firms for each liquidity measure and across the nine measures in the sample. So the study examines how much variation in the unexplained portion of credit spread changes could be explained by the common factors extracted across all the liquidity measures. If the common liquidity factor could explain this portion, then it suggests that the liquidity risk should be incorporated into the pricing of risky debts. We run the panel regressions of the unexplained portion on the first, the second, and the third principal components extracted across various measures, respectively.

\[ \text{Residual}_{ij} = \alpha + \beta \text{Fac}_{ij} + \epsilon_{ij} \]  

(10)

where Fac is the vector of the three common factors extracted from all the liquidity measures including proportion of zero spread changes (Szero), credit market depth (Depth), number of trades in a month (Ntrad), number of days with at least one trade in a month (Nday), effective bid–ask spreads (Bidask), inter-quartile range (IQR), Amihud measure, square root of Amihud measure (S.Amihud), and the range (Range) measure.

As shown in Exhibit 7, the first common factor is significant at the 1% level for the whole sample and the investment-grade sample and at the 5% level for the high-yield sample. For the investment-grade sample, the third common factor is also significant, which suggests that the liquidity common factor has persistent impact on the unexplained part in the credit spreads of the investment-grade firms. The significance level and \( R^2 \) for the investment-grade sample is stronger than that for the high-yield sample. The results are consistent with previous literature which finds that credit spreads of investment-grade firms are less explainable by structural models than high-yield credit spreads.

In addition, the study employs the non-linear regression to estimate the unexplained portion in credit spreads. As shown in Merton [1974], there is a non-linear relation between credit spreads and equity returns. The method could be a robust check for the previous results from linear regressions. The methodology follows Acharya and Johnson [2007], which uses a nonlinear regression to isolate the components in the CDS returns from the fundamental variables suggested by traditional structural models. The article also adds the three-month Treasury bill rates and five-year Treasury rates in the regressions besides those variables used in Acharya and Johnson [2007]. The two Treasury yield

---

**Exhibit 7**

Persistence of Liquidity Measures

<table>
<thead>
<tr>
<th>AutoCorr</th>
<th>Lag1</th>
<th>Lag2</th>
<th>Lag3</th>
<th>Lag4</th>
<th>Lag5</th>
<th>Lag6</th>
<th>Lag7</th>
<th>Lag8</th>
<th>Lag9</th>
<th>Lag10</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.89</td>
<td>0.83</td>
<td>0.75</td>
<td>0.68</td>
<td>0.61</td>
<td>0.53</td>
<td>0.46</td>
<td>0.40</td>
<td>0.33</td>
<td>0.25</td>
</tr>
<tr>
<td>CDS</td>
<td>0.91</td>
<td>0.85</td>
<td>0.73</td>
<td>0.66</td>
<td>0.57</td>
<td>0.49</td>
<td>0.40</td>
<td>0.29</td>
<td>0.20</td>
<td>0.11</td>
</tr>
<tr>
<td>Bond</td>
<td>0.86</td>
<td>0.79</td>
<td>0.72</td>
<td>0.64</td>
<td>0.57</td>
<td>0.49</td>
<td>0.41</td>
<td>0.36</td>
<td>0.30</td>
<td>0.23</td>
</tr>
<tr>
<td>Szero</td>
<td>0.87</td>
<td>0.83</td>
<td>0.70</td>
<td>0.63</td>
<td>0.54</td>
<td>0.46</td>
<td>0.40</td>
<td>0.30</td>
<td>0.21</td>
<td>0.12</td>
</tr>
<tr>
<td>Depth</td>
<td>0.77</td>
<td>0.61</td>
<td>0.45</td>
<td>0.44</td>
<td>0.39</td>
<td>0.35</td>
<td>0.25</td>
<td>0.09</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Ntrad</td>
<td>0.48</td>
<td>0.36</td>
<td>0.32</td>
<td>0.27</td>
<td>0.26</td>
<td>0.23</td>
<td>0.20</td>
<td>0.18</td>
<td>0.33</td>
<td>0.21</td>
</tr>
<tr>
<td>Ndays</td>
<td>0.88</td>
<td>0.78</td>
<td>0.66</td>
<td>0.57</td>
<td>0.50</td>
<td>0.43</td>
<td>0.35</td>
<td>0.27</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td>Bidask</td>
<td>0.90</td>
<td>0.81</td>
<td>0.71</td>
<td>0.62</td>
<td>0.53</td>
<td>0.44</td>
<td>0.35</td>
<td>0.27</td>
<td>0.18</td>
<td>0.08</td>
</tr>
<tr>
<td>Iqr</td>
<td>0.89</td>
<td>0.81</td>
<td>0.73</td>
<td>0.63</td>
<td>0.54</td>
<td>0.47</td>
<td>0.39</td>
<td>0.29</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td>Amihud</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td>S.Amihud</td>
<td>0.77</td>
<td>0.70</td>
<td>0.66</td>
<td>0.54</td>
<td>0.48</td>
<td>0.39</td>
<td>0.34</td>
<td>0.27</td>
<td>0.20</td>
<td>0.13</td>
</tr>
<tr>
<td>Range</td>
<td>0.28</td>
<td>0.21</td>
<td>0.14</td>
<td>-0.01</td>
<td>-0.03</td>
<td>-0.07</td>
<td>-0.04</td>
<td>-0.06</td>
<td>-0.04</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Notes: This exhibit reports the autocorrelation of the first principal component of each liquidity measure and across measures. The common factors are extracted by using the APC method. “All” refers to the first component extracted from all the nine measures. “CDS” refers to the first component extracted from the two credit liquidity measures. “Bond” refers to the first component extracted from all the seven liquidity measures.
variables are also added in the estimation of the unexplained portion in Acharya, Schaefer, and Zhang [2007], which examines the liquidity and correlation risk in the downgrade case of General Motors and Ford in May 2005.

The liquidity measures for each firm are in monthly time series, so the study estimates the monthly residuals for each firm. Let \( \text{CDS}_{it} \) be the credit spreads for firm \( i \) at month \( t \), \( \text{CDSRet}_{it} \) be the monthly CDS spread changes in percentage for firm \( i \) at month \( t \), \( \text{ret}_{it} \) be the monthly equity return for firm \( i \) at month \( t \), \( \Delta r_{3t} \) be the changes in three-month Treasury bill rates, \( \Delta r_{5t} \) be the changes in five-year Treasury yields, then the monthly residuals (unexplained portion in credit spreads) are estimated as the residuals from the following specification:

\[
\text{CDSRet}_{it} = \alpha_i + \sum_{k=1}^{2} [\beta_{i,j;k} + \gamma_{i,j;k} / \text{CDS}_{it}] \text{ret}_{it-k} + \sum_{k=1}^{2} \delta_{i,j;k} \text{CDS}_{it-j,k} + \theta \Delta r_{3t} + \lambda \Delta r_{5t} + \epsilon_{it},
\]

where for each firm, the regression is run for the monthly CDS changes in percentage on a constant, the two lags of the monthly CDS changes, the contemporaneous stock return and its two lags, the ratio of stock returns to the CDS level and its two lags, and the changes in Treasury yields. Acharya and Johnson [2007] used the five lags because they examine whether the degree of asymmetric information adversely affects prices in either equity or credit markets with daily data. In our analysis, the Akaike Information Criterion (AIC) test shows that the regressions including two lags usually have the smallest AIC across all the firms in our sample. Thus, we take the two lags in our tests. The residuals \( \epsilon_{it} \) are regarded as the innovation in the credit market, which are isolated from the news in the stock returns and Treasury yields. Then we examine the liquidity impact on the unexplained portion in credit spreads estimated from the non-linear regressions:

\[
\text{Residual}_{it} = \alpha + \beta \text{Fac}_{it} + \epsilon_{it}
\]

where \( \text{Fac} \) is the vector of the three common factors extracted from all the liquidity measures.

As shown in Exhibit 9, all the first three factors are significant at the 5% level for the whole sample, which shows the strong impact of the common liquidity factor on the unexplained portion in the credit spreads by structural models. The significance does not decrease from the first to the third common factor, which is consistent with the previous results. The results are consistent with Exhibit 8, which suggests that the liquidity common factor could help explain a certain amount of the unexplained portion in credit spreads by the default risk factors implied from structural models.

### Exhibit 8

**Liquidity Impact on Residuals from Linear Regressions**

<table>
<thead>
<tr>
<th></th>
<th>Fac1</th>
<th>Fac2</th>
<th>Fac3</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.0098</td>
<td>-0.0232</td>
<td>-0.0043</td>
<td>-0.0032</td>
</tr>
<tr>
<td></td>
<td>(-1.36)</td>
<td>(-3.14)***</td>
<td>(-0.60)</td>
<td>(-0.44)</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>-0.0052</td>
<td>-0.0165</td>
<td>0.0003</td>
<td>-0.0057</td>
</tr>
<tr>
<td></td>
<td>(-1.79) *</td>
<td>(-5.54)***</td>
<td>(-0.01)</td>
<td>(-1.91) *</td>
</tr>
<tr>
<td>High Yield</td>
<td>-0.0156</td>
<td>-0.0310</td>
<td>-0.0079</td>
<td>-0.0015</td>
</tr>
<tr>
<td></td>
<td>(-1.05)</td>
<td>(-2.02)***</td>
<td>(-0.54)</td>
<td>(-0.10)</td>
</tr>
</tbody>
</table>

**Notes:** The monthly residuals are estimated from the linear regressions using variables implied from structural models. The regression takes the form as:

\[
\Delta \text{CDS} = \alpha + \beta_{ret} + \beta_{lev} + \beta_{vol} + \beta_{yie} + \beta_{dope} + \beta_{mkt} + \beta_{smb} + \beta_{hml} + \beta_{vix} + \epsilon
\]

The panel regressions of the monthly residuals are run on the first three common factors. The regressions are specified as:

\[
\text{Residual}_{it} = \alpha + \beta \text{Fac}_{it} + \epsilon_{it}
\]
E X H I B I T  9  
Liquidity Impact on Residuals from Non-Linear Regressions

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Fac1</th>
<th>Fac2</th>
<th>Fac3</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>-0.0002</td>
<td>-0.0041</td>
<td>-0.0107</td>
<td>-0.0039</td>
</tr>
<tr>
<td></td>
<td>(-0.01)</td>
<td>(-2.18)**</td>
<td>(-5.85)**</td>
<td>(-2.08)**</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>-0.0001</td>
<td>-0.0044</td>
<td>-0.0130</td>
<td>-0.0068</td>
</tr>
<tr>
<td></td>
<td>(-0.01)</td>
<td>(-1.61)</td>
<td>(-4.73)**</td>
<td>(-2.48)**</td>
</tr>
<tr>
<td>High Yield</td>
<td>-0.0003</td>
<td>-0.0036</td>
<td>-0.0087</td>
<td>-0.0007</td>
</tr>
<tr>
<td></td>
<td>(-0.10)</td>
<td>(-1.37)</td>
<td>(-3.62)**</td>
<td>(-0.26)</td>
</tr>
</tbody>
</table>

Notes: The monthly residuals are estimated from the non-linear regressions as:

\[
CDS_{ret,i} = \alpha + \sum_{k=1}^{3} \beta_{k} CDS_{ret,k,i} + \sum_{k=1}^{3} \delta_{k} CDS_{ret,i} + \theta_{k} \Delta r_{k} + \lambda \Delta v_{k} + \varepsilon_{i},
\]

where Fac is the vector of the three common factors extracted from all the liquidity measures. The t-values are in parentheses. ***, ** and * correspond to the significance levels 1%, 5%, and 10%, respectively.

C O N C L U S I O N

The article analyzes two CDS market liquidity measures, proportion of zero daily spread changes and market depth, and seven bond market liquidity measures of trading frequency, trading costs, and trading prices. From the factor decomposition analysis, the article finds a strong commonality across all the various liquidity measures in the fixed-income markets, which is consistent with the findings of the stock market liquidity literature. The unexplained portion in credit spreads by default risk factors are estimated from both linear and nonlinear regressions. The article shows that the liquidity common factors could help explain a certain amount of component in the credit spread change which could not be attributed to the factors of default risk.

E N D N O T E

1Other empirical tests for different variations of the structural models, such as Anderson and Sundaresan [2000] and Eom, Helwege, and Huang [2004], do not find consensus supportive results.

R E F E R E N C E S


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